

# A Comparative Study of Four Open-Ended Coaxial Probe Models for Permittivity Measurements of Lossy Dielectric/Biological Materials at Microwave Frequencies

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**Abstract**—A comparative study of four open-ended coaxial probe models which relate the coaxial line end impedance to the complex permittivity of the material under test is presented. The accuracy of the models in measuring lossy dielectric/biological material and their robustness as a function of the calibration materials are investigated. The four open-ended coaxial probe models studied are: capacitive model, antenna model, virtual line model, and rational function model. Experimental results taken on saline solutions as lossy materials are obtained for the four models.

## I. INTRODUCTION

IN SEVERAL biomedical applications, it is necessary to know the dielectric properties of specific human tissues at microwave frequencies, particularly for hyperthermia treatments. These biological tissues are essentially constituted of water, free ions like  $\text{Na}^+$ ,  $\text{K}^+$ ,  $\text{Ca}^{++}$ ,  $\text{Cl}^-$ , and a great variety of proteins. This composition makes the dielectric properties of the tissues similar to those of saline solutions. These kinds of solutions are characterized by an important dielectric loss factor at low frequencies (less than 3 GHz).

The commonly used technique to obtain the complex dielectric permittivity ( $\epsilon^*$ ) consists of measuring the complex reflection coefficient ( $\Gamma^*$ ) using a network analyzer. Afterwards,  $\epsilon^*$  is obtained through a model that gives the complex admittance of the probe's tip as a function of the dielectric permittivity of the surrounding medium, which is considered semi-infinite.

A certain number of these models exist in the literature [1], [2], [4], and [7]–[10]. However, no comparative experimental studies are available in the open literature to assess the precision of these models, when measurements are made on high loss media such as saline solutions or biological tissues. The main goal of this article is to assess the robustness of each model when high loss materials are measured. Consequently, a judicious choice of the open-ended coaxial probe model can be made when measurements of biological tissues are required.

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## II. EXPERIMENTAL SETUP FOR DIELECTRIC MEASUREMENTS

All the measurements of the complex reflection coefficient used in this paper were obtained with a network analyzer (AP-8510) between 50 MHz and 20.05 GHz with a 50 MHz step (401 points). The calibration of the network analyzer was done using the OSL technique (open, short, and matched load connected at the end of the analyzer's cable). The probe was fabricated from a semirigid 50  $\Omega$  coaxial line with outer diameter of 2.2 mm and inner diameter of 0.65 mm. The coaxial probe's end was immersed in the dielectric under test and the reflection coefficients were recorded at room temperature (25°C). The timegating option of the network analyzer was also used to remove the effects of the discontinuities between the reference plane (defined at the probe's connector) and the end of the probe. This option calculates the inverse Fourier transform (IFFT) of the complex reflection coefficient at the reference plane in order to obtain the temporal response of the probe. Afterwards, one has to select an observation window, in the time domain, containing only the reflection at the probe's end to reject all of the parasitic reflections. The network analyzer calculates the Fourier transform (FFT) of this window and convolves the result with the original measured reflection coefficient at reference plane. The obtained effect, in the frequency domain, is a smoothing of the initial curve where the effects of the discontinuities are present.

## III. MODELING OF THE OPEN-ENDED COAXIAL PROBE

Four models have been considered for this study: the capacitive model [1] and [2], the antenna model [4], the virtual line model [7], and the rational function model [8]–[10]. The following sections present a brief description of these models.

### A. Capacitive Model

The equivalent circuit for this model is presented in Fig. 1(b). The model is described in [1] and [2]. The reflection coefficient  $\Gamma^*$  at the tip of the open-ended probe is obtained by considering the complex admittance of the equivalent circuit

$$\Gamma^* = \Gamma^{\text{J}\Phi} = \frac{1 - j\omega Z_0(C(\epsilon^*) + C_f)}{1 + j\omega Z_0(C(\epsilon^*) + C_f)} \quad (1)$$

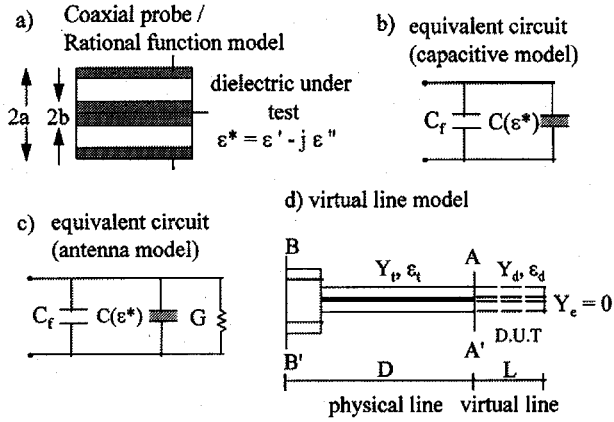


Fig. 1. Schema of the four models used in this study.

where:

$$\begin{aligned} C(\epsilon^*) &= \epsilon^* C_0 \\ \omega &\text{ is the angular frequency.} \\ Z_0 &\text{ is the characteristic impedance of the transmission line.} \end{aligned}$$

Solving for  $\epsilon^*$ , one can obtain

$$\epsilon^* = \frac{1 - \Gamma^*}{j\omega Z_0 C_0 (1 + \Gamma^*)} - \frac{C_f}{C_0}. \quad (2)$$

It is necessary to calculate the value of the two unknown variables  $C_f$  and  $C_0$  to be able to find  $\epsilon^*$ . In order to do this, the use of a calibration medium for which the dielectric properties are well known is required. A frequently used medium is deionized water. The two unknown parameters are then given by the following equations:

$$C_0 = \frac{(1 - |\Gamma_{diel}^*|^2)}{\omega Z_0 (1 + 2|\Gamma_{diel}^*| \cos(\Phi_{diel}) + |\Gamma_{diel}^*|^2) \epsilon''} \quad (3a)$$

and

$$C_f = \frac{-2|\Gamma_{diel}^*| \sin(\Phi_{diel})}{\omega Z_0 (1 + 2|\Gamma_{diel}^*| \cos(\Phi_{diel}) + |\Gamma_{diel}^*|^2)} - \epsilon'_{diel} C_0 \quad (3b)$$

where:

$\Gamma_{diel}^*$  is the complex reflection coefficient of the dielectric calibration medium referred at the end of the probe;  
 $\Phi_{diel}$  is the phase of  $\Gamma_{diel}^*$ ;

and

$\epsilon'_{diel}$  is the real part of the complex permittivity of the calibration medium.

After the calculation of these parameters, we are able to measure the complex dielectric permittivity of a medium. However, if the reference plane has previously been defined at the entrance of the probe ( $B - B'$  plane), the measured  $\Gamma^*$  will be referred at this plane and not at the end of the probe ( $A - A'$  plane). Since this model requires a  $\Gamma^*$  value referred at the  $A - A'$  plane, one must find the phase difference between the  $B - B'$  and  $A - A'$  planes. The reflection coefficients relative to these two planes are related in this way

$$\Gamma_{A-A'}^* = \Gamma_{B-B'}^* e^{j2\theta} \quad (4)$$

where the  $2\theta$  factor in the exponent is necessary to take into account of the round-trip of the wave in the probe. This factor is calculated in this way

$$2\theta = \Phi_{A-A'} - \Phi_{B-B'} \quad (5)$$

where

$$\Phi_{A-A'} = \angle \Gamma_{A-A'}^*$$

and

$$\Phi_{B-B'} = \angle \Gamma_{B-B'}^*.$$

The phase  $\Phi_{B-B'}$  is easily measurable with the network analyzer but the phase of the reflection coefficient referred at  $A - A'$  ( $\Phi_{A-A'}$ ) is not directly measurable. To calculate the value of  $2\theta$ , it is necessary to use a model which gives the value of  $\Phi_{A-A'}$ .

The determination of the round-trip phase factor  $2\theta$  was made by measuring the complex reflection coefficient in air  $\Gamma_{B-B'}^*(\text{air})$  as follows: the time-gating option of the ANA was used to remove the parasitic reflections and the reflection coefficient  $\Gamma_{B-B'}^*(\text{air})$  was measured at the desired frequencies. Afterwards, the model giving the value of  $\Phi_{A-A'}$  was used to calculate the  $2\theta$  factor.

The literature (see [3]) gives a theoretical expression for  $C_f + C_0$  when the probe's end is in the air (open-circuit)

$$C_f + C_0 = 2.38\epsilon_0(b - a) \quad (6)$$

where:

$a$  is the radius of internal conductor; and  
 $b$  is the radius of external conductor.

Inserting (6) in (1) we can obtain the following relationship giving the value of the complex reflection coefficient  $\Gamma_0^*$  at the end of the probe ( $A - A'$  plane) as follows:

$$\Gamma_0^* = \frac{1 - j \cdot 2.38\omega Z_0 \epsilon_0 (b - a)}{1 + j \cdot 2.38\omega Z_0 \epsilon_0 (b - a)}. \quad (7)$$

The value of  $\Phi_{A-A'}$  is obtained by calculating the phase of  $\Gamma_0^*$

$$\Phi_0 = -2.38\omega Z_0 \epsilon_0 (b - a). \quad (8)$$

And the value of  $2\theta$  is simply obtained by the following equation:

$$2\theta = -4.76\omega Z_0 \epsilon_0 (b - a) - \Phi_{B-B'}. \quad (9)$$

The reflection coefficient obtained with the network analyzer can thus be de-embedded to the end of the probe. It is important to note that all the values of  $\Gamma^*$  must be multiplied by the  $e^{j2\theta}$  factor for the calculation of  $C_f$ ,  $C_0$  and  $\epsilon^*$ .

## B. Antenna Model

This model is given in reference [4]. The probe is modeled by a capacitance  $C_f$ , while the liquid is modeled by a capacitance  $\epsilon^* C_2$  and a resistor  $R$  (inverse of the conductance  $G$ ) connected in parallel to the capacitances (see Fig. 1(c)). The admittance of this equivalent circuit is given by

$$\frac{Y}{Y_0} = j\omega C_1 Z_0 + j\omega C_2(\omega, \epsilon^*) + Z_0 G(\omega, \epsilon^*) \quad (10)$$

where:

- $Z_0$  is the characteristic impedance of the coaxial line (50  $\Omega$ );  
 $Y_0$  is the admittance of the coaxial line ( $1/Z_0$ );  
 $\omega = 2\pi f$ , and  
 $\epsilon^* = \epsilon' - j\epsilon''$  is the complex permittivity of the surrounding medium.

A coaxial probe immersed in a lossy medium will radiate in that medium: the probe can thus be considered like a functional antenna. When the probe is considered to be infinitesimal, the power radiated from the end of the coaxial line in free space is [5]

$$P_r = G \cdot V^2 = \frac{16\pi^5 \omega^4}{3\eta_0 c^4} \left[ \frac{b^2 - a^2}{\log(b/a)} \right]^2 \quad (11)$$

where:

- $V$  is the voltage between the inner and outer conductor at the open end;  
 $\omega$  is the angular frequency;  
 $b$  and  $a$  are the radius of the outer and inner conductor, respectively;  
 $\eta_0$  is the complex intrinsic impedance of the free space  
 $(\eta_0 = (\mu_0/\epsilon_0)^{1/2})$  and  
 $c$  is the speed of the light in free space.

For an antenna in a nonmagnetic lossy medium, it has been shown that [6]

$$Y^*(\omega, \epsilon^*) = \sqrt{\epsilon^*} Y(\sqrt{\epsilon^*} \omega, \epsilon_0) \quad (12)$$

where:

- $Y^*$  is the admittance of the antenna in the nonmagnetic lossy medium;  
 $Y$  is the admittance of the antenna in free space;  
 $\epsilon^*$  is the complex permittivity of the nonmagnetic lossy medium; and  
 $\epsilon_0$  is the permittivity of free space.

Inserting (11) in (12) and replacing the result in (10), it is possible to deduce

$$\frac{Y}{Y_0} = j\omega C_1 Z_0 + j\omega \epsilon^* C_2 Z_0 + \epsilon^{*5/2} G Z_0. \quad (13)$$

Equation 13 has the following form:

$$\frac{Y}{Y_0} = K_1 + K_2 \epsilon^* + K_3 \epsilon^{*5/2}. \quad (14)$$

The factors  $K_1$ ,  $K_2$ , and  $K_3$  are generally complex. To determine these factors, one must use three media for which the dielectric permittivities are well known. The media that have been used for the determination of the three unknown parameters are: deionized water, methanol, and air. When these parameters are determined, the calculation of  $\epsilon^*$  of the unknown medium can be done. Since a five-order complex equation (14) has to be solved to find the complex permittivity of the unknown medium, one must select an appropriate solution ( $\epsilon' > 1$  and  $\epsilon'' < 0$ ). In all cases investigated in this study, only one solution had a physical sense.

To correctly use this model, the measured complex admittance ( $Y$ ) has to be referred to the end of the probe (plane  $A - A'$ ). In order to do that, the procedure given at Section III-A [(4) to (8)] can be used.

### C. Virtual Line Model

This model has been developed in the end of the 1980s [7]. It consists of modeling the dielectric medium by a virtual transmission line of length  $L$  which has the same dimensions as the physical line. The virtual transmission line is terminated by an open circuit, this is shown in Fig. 1(d). The complex admittance at  $A - A'$  plane is given by this relation

$$Y_L = Y_d \frac{Y_E + jY_d \tan(\beta_d L)}{Y_d + jY_E \tan(\beta_d L)} \quad (15)$$

where:

- $Y_L$  is the admittance at the input of the virtual transmission line;  
 $Y_d$  is the characteristic admittance of the virtual transmission line;  
 $Y_E$  is the terminating admittance at the end of the virtual transmission line;  
 $\beta_d$  is the propagation constant in the test medium; and  
 $L$  is the virtual transmission line length.

Since the virtual line is terminated by an open circuit,  $Y_E = 0$ , (15) becomes

$$Y_L = jY_d \tan(\beta_d L). \quad (16)$$

The virtual line is coaxial, therefore, its characteristic admittance is

$$Y_d = \frac{\sqrt{\epsilon_d}}{60 \ln(b/a)} \quad (17)$$

where:

- $b$  is the external diameter of the line;  
 $a$  is the internal diameter of the line; and  
 $\epsilon_d$  is the complex permittivity of tested medium.

In addition, the characteristic admittance at the input of the probe can be referred to the input of the virtual line as follows:

$$Y_L = \frac{1 - \Gamma_m e^{2j\beta_t D}}{1 + \Gamma_m e^{2j\beta_t D}} Y_t \quad (18)$$

where:

- $Y_t$  is the admittance of the coaxial probe;  
 $\beta_t$  is the propagation constant of the coaxial probe; and  
 $\Gamma_m$  is the complex reflection coefficient measured at plane  $B - B'$ .

Inserting (18) and (17) in (16) and solving for the permittivity of the tested medium, one can obtain

$$\epsilon_d = \frac{-jc\sqrt{\epsilon_t}}{2\pi fL} \cdot \frac{1 - \Gamma_m e^{2j\beta_t D}}{1 + \Gamma_m e^{2j\beta_t D}} \cot\left(\frac{2\pi fL\sqrt{\epsilon_d}}{c}\right). \quad (19)$$

In this equation, there are two unknown variables: the length of the physical line ( $D$ ) and the length of the virtual line ( $L$ ) which cannot be directly measured since its existence is hypothetical and constitutes the basis of this model. These two

unknowns are obtained from the reflection coefficients measured with two calibration media with known permittivities (air and deionized water, or air and methanol) through an iterative procedure described in detail in [7]. It is interesting to note that all complex reflection coefficients used in this model are referred to the input of the probe ( $B - B'$  plane).

#### D. Rational Function Model

This model has been developed in the early 1990s (see [8]–[10]). The complex admittance of a 50  $\Omega$  coaxial probe immersed in a dielectric medium has been computed with the moment method. The geometry of the problem is shown in Fig. 1(a). The results obtained with this method include the radiation effects, the energy storage in the near field region and the evanescent mode of the guide. The model is described by this equation

$$\frac{Y}{Y_0} = \frac{\sum_{n=1}^4 \sum_{p=1}^8 \alpha_{np} (\sqrt{\epsilon^*})^p (j\omega a)^n}{1 + \sum_{m=1}^4 \sum_{q=0}^8 \beta_{mq} (\sqrt{\epsilon^*})^q (j\omega a)^m} \quad (20)$$

where:

- $\alpha_{np}$  and  $\beta_{mq}$  are the coefficients of the model (given in [10]);
- $\epsilon^*$  is the complex permittivity of the tested dielectric;
- $a$  is the inner conductor radius of the line;
- $Y$  is the admittance at the end of the coaxial probe; and
- $Y_0$  is the characteristic admittance of the coaxial probe.

This model is valid if the relative permittivity of the tested medium and the frequency are in these ranges: ( $1 \leq \epsilon' \leq 80$ ), ( $-80 \leq \epsilon'' \leq 0$ ), and ( $1 \leq f \leq 20$  GHz).

Equation 20 gives the value of the complex admittance at the end of the probe as a function of the complex permittivity of the tested medium and the probe's dimensions. The inverse problem, which consists of calculating the complex permittivity constant of the dielectric medium under test from the measured complex admittance, is solved as follows [9]:

$$\sum_{i=0}^8 (b_i - Y c_i) \sqrt{\epsilon^*}^i = 0 \quad (21)$$

where

$$b_p = \sum_{m=1}^4 \alpha_{mp} (j\omega a)^m \quad p = 1, 2, \dots, 8;$$

$$b_0 = 0;$$

$$c_q = \sum_{m=1}^4 \beta_{mq} (j\omega a)^m \quad q = 1, 2, \dots, 8$$

and

$$c_0 = 1 + \sum_{m=1}^4 \beta_{m0} (j\omega a)^m.$$

Here again, it is necessary to use complex admittance values referred to the end of the probe, so, we can use the same method as given in Section III-A. Since an eight-order complex equation (21) has to be solved to find the complex permittivity of the tested medium, an appropriate solution must be selected

( $\epsilon' > 1$  and  $\epsilon'' < 0$ ). In all cases investigated in this study, only one solution had a physical sense.

An interesting characteristic of this model is that we do not need to find any calibration parameters and, consequently, we do not need to use any standard dielectric media. Indeed, the parameters  $\alpha_{np}$  and  $\beta_{mq}$  have been established and optimized by using 56 dielectric media included in this range:  $1 \leq \epsilon' \leq 80$ .

#### IV. EXPERIMENTAL RESULTS ON SALINE SOLUTIONS

This part presents the measured results (between 1 GHz and 20 GHz) for two saline solutions. The concentration of these solutions was 0.5 mol/L and 1.0 mol/L. The choice of saline solutions comes from the fact that their properties are similar to those of biological tissues. However, the losses of these chosen saline solutions are habitually higher than those of normal biological tissues, especially for the 1.0 mol/L solution at low frequencies. This choice was made to test the accuracy of the four models in extreme conditions in order to know their degree of reliability with respect to the loss factor. Indeed, we can assume that a model which can offer the best accuracy under very high loss conditions can also offer a good accuracy under normal loss condition. When permittivity is measured on an unknown lossy dielectric or biological tissue, we want to be sure that the used model can give accurate results under a wide range of loss factor. These reasons explain the choice of the saline solutions concentration.

It has been demonstrated in the previous sections, that some models require measurements of known media to find the value of unknown calibration parameters. In the following results, we have used these liquids as calibration media:

Model	Calibration Media
capacitive <sup>1</sup>	deionized water
antenna <sup>2</sup>	air, deionized water, methanol
virtual line <sup>3</sup>	air, deionized water
rational function <sup>4</sup>	none

The theoretical dielectric properties of NaCl(aq) solutions [11] are shown in Fig. 2 while the obtained results are given in Figs. 3 and 4. According to these results, one can make the following statements:

For NaCl(aq) [0.5M]

- 1) The best results are obtained by the virtual line model and the antenna model. The obtained results for  $\epsilon''$  are accurate while those obtained for  $\epsilon'$  are acceptable.
- 2) The results obtained by the rational function method are good when the frequency is between 2 and 5 GHz. However, discrepancies between measured and reference values increase with the frequency.
- 3) The capacitive model does not give accurate results for low frequencies, especially for  $\epsilon''$ .

<sup>1</sup> After de-embedding procedure using air [see Section III-A, (4)–(9).]

<sup>2</sup> After de-embedding procedure using air [see Section III-A, (4)–(9).]

<sup>3</sup> The de-embedding procedure is included in the model (parameter D).

<sup>4</sup> After de-embedding procedure using air [see Section III-A, (4)–(9).]

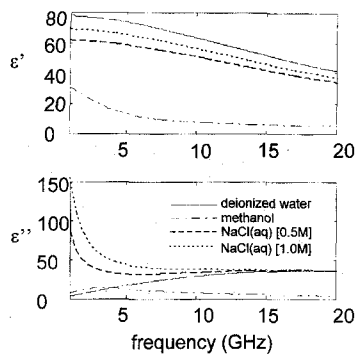


Fig. 2. Dielectric properties of deionized water, methanol and NaCl(aq) (0.5M and 1.0M).

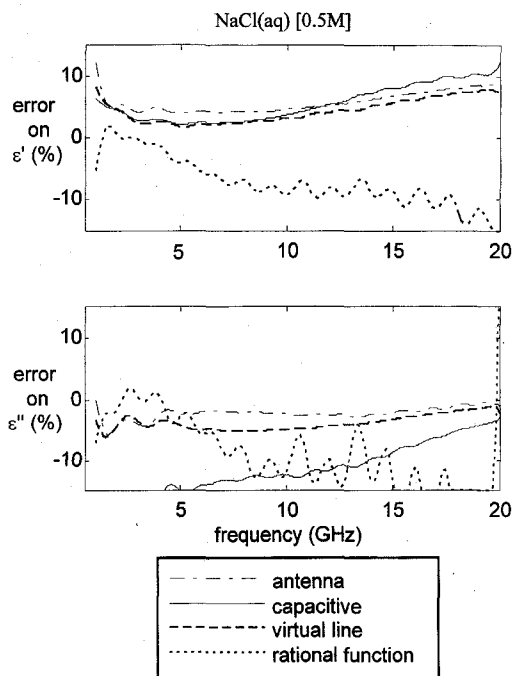


Fig. 3. Relative error of models on dielectric properties of NaCl(aq) [0.5M].

For NaCl(aq) [1.0M]

- 1) The virtual line model and the antenna model give the best results for  $\epsilon''$ . However, the error on  $\epsilon'$  obtained with the antenna model is very significant at low frequencies (below 2 GHz).
- 2) For  $\epsilon'$ , the best results are obtained with the virtual line and the rational function models.
- 3) The rational function model gives excellent results between 2 and 5 GHz. However, outside of this band, the error has a tendency to become significant, for both  $\epsilon'$  and  $\epsilon''$ .
- 4) The capacitive model does not give satisfying results: the error on  $\epsilon'$  increases with frequency but, conversely, error on  $\epsilon''$  decreases with frequency.

#### V. SENSITIVITY TO THE CALIBRATION MEDIA OF THE VIRTUAL LINE MODEL AND THE CAPACITIVE MODEL

The previous section shows that results obtained with the virtual line model are accurate for high loss saline solutions.

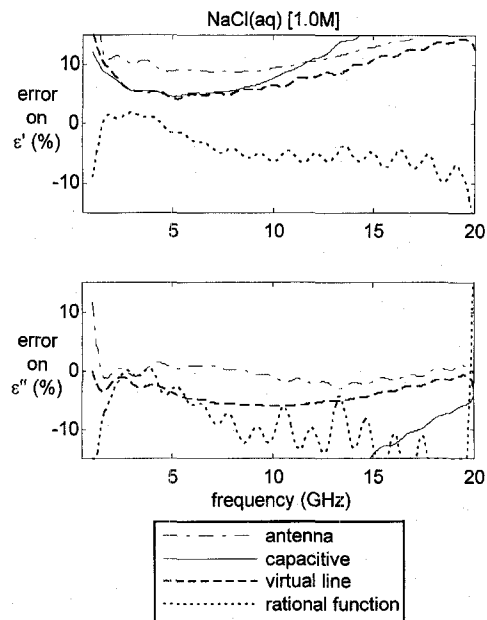


Fig. 4. Relative error of models on dielectric properties of NaCl(aq) [1.0M].

However, this model requires two calibration media to determine the two unknown parameters ( $L$  and  $D$ ). One can ask the following question: What will the effects of the selection of the calibration media be on the overall accuracy of the measured complex permittivities? Or, in other words, what is the robustness of this model with respect to the select calibration media? Indeed, one can assume that the closer the dielectric properties of the calibration media are to the medium under test, the more accurate will be the results.

To answer these questions, complex permittivity measurements have been performed on NaCl(aq) [0.5M] solutions using different calibration media. Comparisons between the virtual line model and the well-known capacitive model have been made. The rational function model was not considered in this section since no calibration media are required. The antenna model was not considered since three dielectric media are required for the calibration step. Indeed, deionized water and methanol were used because their dielectric properties are well known and high chemical purity samples are easily obtainable. However, experiment has shown that the errors on measured permittivity become very significant when another calibration medium is used (such as ethanol or propanol). This can be due to the low purity of the calibration medium or to the poor accuracy of the theoretical dielectric properties given in the literature. Since only air, deionized water, and methanol can be used as reliable calibration standards, we were not able to study the effects of the selection of another calibration liquid on the permittivity measurement for the antenna model.

First, for the two considered models, deionized water was chosen as calibration medium because its dielectric properties are similar to those of saline solutions. Second, methanol was chosen for its very different dielectric properties. The calibration media were as shown in Table I.

The dielectric properties of deionized water, methanol and NaCl(aq) [0.5M] are given in Fig. 2 while the obtained results

TABLE I

	virtual line model	capacitive model <sup>5</sup>
Fig. 5	deionized water, air	deionized water
Fig. 6	methanol, air	methanol

<sup>5</sup>After the de-embedding procedure using air [see Section III-A, (4)–(9).]

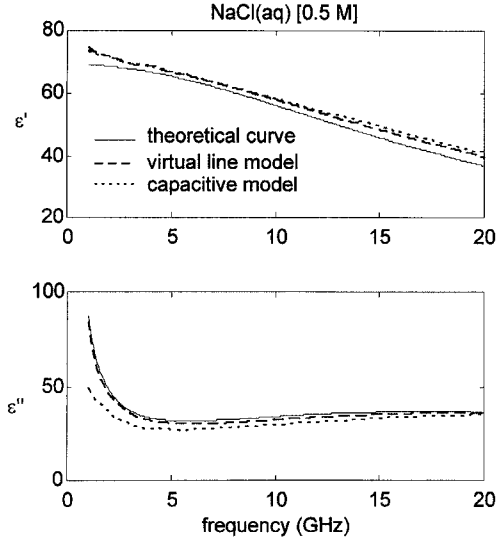


Fig. 5. Permittivity of NaCl(aq) [0.5M] with water and air as calibration media.

are given in Figs. 5 and 6. We can see that the virtual line model gives good results when deionized water is used as calibration medium while the capacitive model provides erroneous results at low frequencies for  $\epsilon''$ . However, these two models give inaccurate results when methanol is used as the calibration medium. On the other hand, one can state that the virtual line model is less sensitive to the calibration medium than the capacitive model, especially for low frequencies. The  $L$  parameter of the virtual line model (like the  $C_f$  and  $C_0$  parameters of capacitive model) is dependent on the calibration media. Nevertheless, when the dielectric properties of the calibration medium are close to those of the dielectric under test, the results provided by the virtual line model are very accurate. It is thus recommended, when the virtual line model is to be used, that deionized water and air should be used as the calibration media.

## VI. DISCUSSION

The model which overall gives the best results for permittivity measurements of high loss solutions is the virtual line one. The results obtained with the antenna model are also accurate except at low frequencies. The rational function model can be used if its validity range is respected ( $1 < f < 20$  GHz,  $1 < \epsilon' < 80$ ,  $-80 < \epsilon'' < 0$ ). However, when the NaCl concentration is higher than 0.5 mol/L, the imaginary part of  $\epsilon^*$  can be less than  $-80$  at low frequencies, which

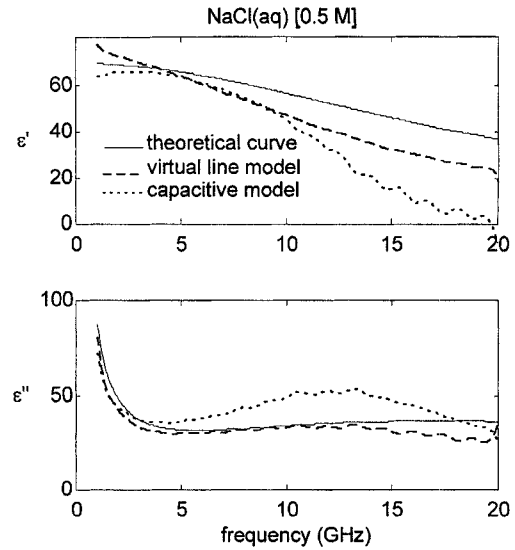


Fig. 6. Permittivity of NaCl(aq) [0.5M] with methanol and air as calibration media.

is out of the validity range of the model. When permittivity measurements of high concentration saline solutions are made, the model may be at the limit of its validity, especially at low frequencies. It is important to state that coefficients used in this study (given in [10]) are not completely suitable for high loss dielectric measurements. However, it is expected that a new set of coefficient could be obtained for high loss dielectric measurements, which is beyond of the scope of this paper. For the evaluation of the capacitive model, we can say that it does not give good results for permittivity measurement of saline solutions. The fact that the antenna model gives better results than this one is caused by the presence of a resistor in parallel with the two capacitances which models the losses in the medium. The deterioration of the antenna model, at low frequencies, can be explained by the fact that the radiation efficiency of the probe decreases with frequency: the probe no longer behaves like an antenna. This model does not represent adequately the physical situation at low frequencies and the results obtained with it are not very accurate. Regarding the virtual line model, the precision obtained in the studied cases is acceptable. This can be explained by the fact that the physical propagation phenomenon in the medium is modeled by the virtual transmission line. Even if this model assumes a perfect TEM propagation mode and neglects all of the high order modes, the obtained results show that it offers relatively good precision. Experimental tests have shown that the precision of the results are dependent on the calibration media: accuracy is high when the dielectric properties of the calibration and tested medium are close. Nevertheless, the model is robust enough to allow some freedom in the selection of the calibration media. The use of deionized water as the calibration medium gives good results when the permittivity of a saline solution is measured. This technique seems to overestimate a little bit the real part of permittivity when the frequency is below 2 GHz. This might be caused by the fact that the dielectric properties of water and saline solution are quite different at

low frequencies: the losses are high for the saline solution and low for water.

## VII. CONCLUSION

The accuracy of four open-ended coaxial probe models was investigated. It has been found that the virtual line model was well suited for complex permittivity measurements of high loss media such as biological tissues. This model is sufficiently robust to achieve precise results when deionized water and air are used as calibration media. The antenna model can give accurate values on  $\epsilon''$  for lossy materials. However, its accuracy for  $\epsilon'$  is poor, especially when the losses are important. The rational function model is interesting since no calibration medium is required providing that the model parameters are obtained using a set of high loss dielectric media. Using the published parameters, its accuracy was found to be not acceptable when high losses dielectric properties are measured. Regarding the capacitive model, experiments have shown that this model is not adequate for dielectric property measurements on high loss materials. In conclusion, the results indicate that the use of the virtual line model for biological tissue characterization is adequate and it offers a relatively good confidence for measurement results.

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